Practice Test - Chapter 3

Solve each system of equations by using either substitution or elimination.

1. $y = x + 4$
   $x + y = -12$

SOLUTION:

Substitute $x + 4$ for y in the second equation and solve for x.

$$\begin{aligned} x + x + 4 &= -12 \\ 2x + 4 &= -12 \\ 2x &= -16 \\ x &= -8 \end{aligned}$$

Substitute –8 for x in the first equation and solve for y.

$$\begin{aligned} y &= -8 + 4 \\ &= -4 \end{aligned}$$

Therefore, the solution is $(-8, -4)$.

2. $3x + 5y = -7$
   $6x - 4y = 0$

SOLUTION:

Multiply the first equation by –2 and add with the second equation.

$$\begin{aligned} -6x - 10y &= 14 \\ 6x - 4y &= 0 \\ -14y &= 14 \\ y &= -1 \end{aligned}$$

Substitute –1 for y in the second equation and solve for x.

$$\begin{aligned} 6x - 4(-1) &= 0 \\ 6x &= -4 \\ x &= -\frac{2}{3} \end{aligned}$$

Therefore, the solution is $\left(-\frac{2}{3}, -1\right)$.

3. $5x + 2y = 4$
   $3y - 4x = -40$

SOLUTION:

Multiply the first and the second equation by 4 and 5 then add.

$$\begin{aligned} 20x + 8y &= 16 \\ -20x + 15y &= -200 \\ 23y &= -184 \\ y &= -8 \end{aligned}$$

Substitute –8 for y in the second equation and solve for x.

$$\begin{aligned} 5x + 2(-8) &= 4 \\ 5x - 16 &= 4 \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

Therefore, the solution is $(4, -8)$.

4. $8x - 3y = -13$
   $-3x + 5y = 1$

SOLUTION:

Multiply the first and the second equation by 5 and 3 respectively then add.

$$\begin{aligned} 40x - 15y &= -65 \\ -9y + 15y &= 3 \\ 31x &= -62 \\ x &= -2 \end{aligned}$$

Substitute –2 for x in the second equation and solve for y.

$$\begin{aligned} -3(-2) + 5y &= 1 \\ 6 + 5y &= 1 \\ 5y &= -5 \\ y &= -1 \end{aligned}$$

Therefore, the solution is $(-2, -1)$.

5. **MULTIPLE CHOICE** Which graph shows the solution of the system of inequalities?

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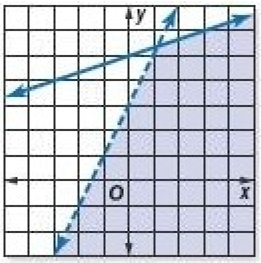
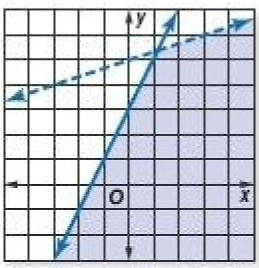
$y \le 2x + 3$

$y < \frac{1}{3}x + 5$

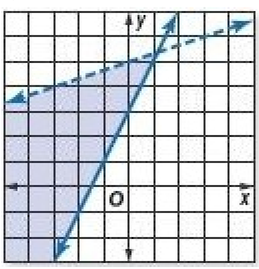
A



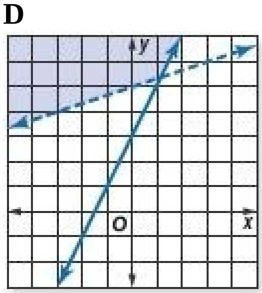
B



C

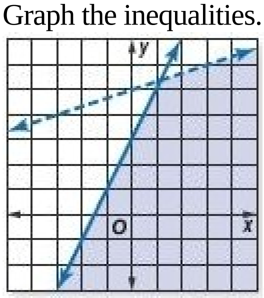


D



SOLUTION:

Graph the inequalities.



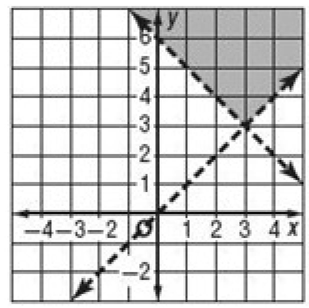
Option B is the correct answer.

Solve each system of inequalities by graphing.

6. $x + y > 6$

$x - y < 0$

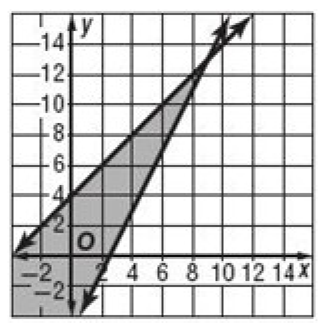
SOLUTION:



7. $y \ge 2x - 5$

$y \le x + 4$

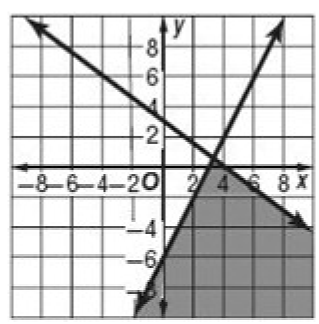
SOLUTION:



8. $3x + 4y \le 12$

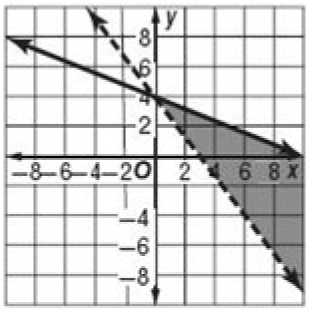
$6x - 3y \ge 18$

SOLUTION:



9. $5y + 2x \le 20$
$4x + 3y > 12$

SOLUTION:

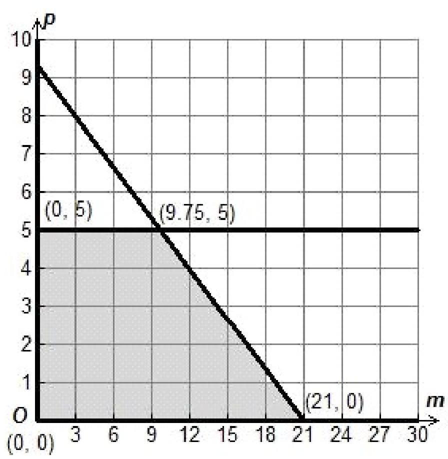


10. **SALONS** Sierra King is a nail technician. She allots 20 minutes for a manicure and 45 minutes for a pedicure in her 7-hour word day. No more than 5 pedicures can be scheduled each day. The prices are \$18 for a manicure and \$45 for a pedicure. How many manicures and pedicures should Ms. King schedule to maximize her daily income? What is her maximum daily income?

SOLUTION:

Let m represent the number of manicures and p represent the number of pedicures.
Write the system of constraints for the scenario. Since she cannot do a negative procedure, both variables must be greater than or equal to 0.
$0 \le p \le 5$
$m \ge 0$
$20m + 45p \le 420$

Graph the system of inequalities and find the vertices of the feasible region.



The vertices of the feasible region are (0 ,0), (0, 5), (9.75, 5), and (21, 0).

Since the price for each manicure is \$18 and the price for each pedicure is \$45, the function that represents her daily income is $I(m,p) = 185m + 45p$. Sierra can only do a whole number of manicures and pedicures so substitute (0, 0), (0, 5), (9, 5), and (18, 0) in the income function to determine the maximum.

(m, p)	$18m + 45p$	$I(m, p)$
(0, 0)	18(0) + 45(0)	0
(0, 5)	18(0) + 45(5)	225
(9, 5)	18(9) + 45(5)	387
(21, 0)	18(21) + 45(0)	378

The maximum income will be produced when she schedules 9 manicures and 5 pedicures.
This will produce a maximum income of \$387.

11. **COLLEGE FOOTBALL** In a recent year, Darren McFadden of Arkansas placed second overall in the Heisman Trophy voting. Players are given 3 points for every first-place vote, 2 points for every second-place vote, and 1 point for every third-place vote. McFadden received 490 total votes for first, second, and third place, for a total of 878 points. If he had 4 more than twice as many second-place votes as third-place votes, how many votes did he receive for each place?

SOLUTION:

Let x, y and z be the number of first, second and third place vote.
The system of equations represent this situation is:

$$x + y + z = 490 \rightarrow (1)$$
$$3x + 2y + z = 878 \rightarrow (2)$$
$$y = 2z + 4 \rightarrow (3)$$

Substitute $2y + 4$ for z in the first and second equation and simplify.

$$x + 2z + 4 + z = 490$$
$$x + 3z = 486 \rightarrow (4)$$

$$3x + 2(2z + 4) + z = 878$$
$$3x + 4z + 8 + z = 878$$
$$3x + 5z = 870 \rightarrow (5)$$

Multiply the fourth equation by –3 and add with the fifth equation.

$$\begin{aligned} &(4)\times -3 \quad -3x - 9z = -1458 \\ &(5) \qquad\quad 3x + 5z = 870 \\ &\qquad\qquad\quad -4z = -588 \\ &\qquad\qquad\qquad z = 147 \end{aligned}$$

Substitute 147 for z in the fourth equation and solve for x.

$$\begin{aligned} x + 3(147) &= 486 \\ x + 441 &= 486 \\ x &= 45 \end{aligned}$$

Substitute 147 for z in the third equation and solve for y.

$$\begin{aligned} y &= 2(147) + 4 \\ &= 294 + 4 \\ &= 298 \end{aligned}$$

Therefore, he received 45 first-place, 298 second-place and 147 third-place votes.

Perform the indicated operations. If the matrix does not exist, write *impossible*.

12. $-3\begin{bmatrix} 4a \\ 0 \\ -3 \end{bmatrix} + 4\begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$

SOLUTION:

Distribute the scalar.

$$\begin{aligned} -3\begin{bmatrix} 4a \\ 0 \\ -3 \end{bmatrix} + 4\begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} &= \begin{bmatrix} -3(4a) \\ -3(0) \\ -3(-3) \end{bmatrix} + \begin{bmatrix} 4(-2) \\ 4(3) \\ 4(-1) \end{bmatrix} \\ &= \begin{bmatrix} -12a \\ 0 \\ 9 \end{bmatrix} + \begin{bmatrix} -8 \\ 12 \\ -4 \end{bmatrix} \end{aligned}$$

Add corresponding elements.

$$\begin{aligned} &= \begin{bmatrix} -12a - 8 \\ 0 + 12 \\ 9 - 4 \end{bmatrix} \\ &= \begin{bmatrix} -12a - 8 \\ 12 \\ 5 \end{bmatrix} \end{aligned}$$

13. $\begin{bmatrix} -3 & 0 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ -6 & 0 \end{bmatrix}$

SOLUTION:

$$\begin{aligned} \begin{bmatrix} -3 & 0 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ -6 & 0 \end{bmatrix} &= \begin{bmatrix} (-3)(2)+(0)(-6) & (-3)(4)+(0)(0) \\ (1)(2)+(5)(-6) & (1)(4)+(5)(0) \end{bmatrix} \\ &= \begin{bmatrix} -6 & -12 \\ -28 & 4 \end{bmatrix} \end{aligned}$$

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14. $\begin{bmatrix} 2 & 0 \\ -3 & 5 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix}$

SOLUTION:

$$\begin{bmatrix} 2 & 0 \\ -3 & 5 \\ 1 & 4 \end{bmatrix}\cdot\begin{bmatrix} 3 \\ -2 \end{bmatrix}=\begin{bmatrix} (2)(3)+(0)(-2) \\ (-3)(3)+(5)(-2) \\ (1)(3)+(4)(-2) \end{bmatrix}$$
$$=\begin{bmatrix} 6 \\ -19 \\ -5 \end{bmatrix}$$

15. $\begin{bmatrix} -5 & 7 \\ 6 & 8 \end{bmatrix}-\begin{bmatrix} 4 & 0 & -2 \\ 9 & 0 & 1 \end{bmatrix}$

SOLUTION:

The inner dimensions are not equal. So, the matrix subtraction is impossible.

16. **MULTIPLE CHOICE** What is the value of $\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix}$?

F –44
G 44
H $\frac{1}{44}$
J $-\frac{1}{44}$

SOLUTION:

$$\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix}=2(12-20)-3(0+8)-1(0+4)$$
$$=2(-8)-3(8)-1(4)$$
$$=-16-24-4$$
$$=-44$$

Therefore, option F is the correct answer.

Find the inverse of each matrix, if it exists.

17. $\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$

SOLUTION:

Let $A=\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$.

det (A) = 5

$$A^{-1}=\frac{1}{5}\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$$
$$=\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & 1 \end{bmatrix}$$

18. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

SOLUTION:

Let $A=\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

det (A) = –3

$$A^{-1}=\frac{1}{-3}\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$$
$$=\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

19. $\begin{bmatrix} 6 & 3 \\ 8 & 4 \end{bmatrix}$

SOLUTION:

Let $A=\begin{bmatrix} -3 & -2 \\ 6 & 4 \end{bmatrix}$.

det (A) = 0
The value of the matrix is zero. So the inverse does not exist.

20. $\begin{bmatrix} -3 & -2 \\ 6 & 4 \end{bmatrix}$

SOLUTION:

Let $A = \begin{bmatrix} 6 & 3 \\ 8 & 4 \end{bmatrix}$.

det (A) = 0

The value of the matrix is zero. So the inverse does not exist.

Use Cramer's Rule to solve the following system of equations.

21. $2x - y = -9$

$x + 2y = 8$

SOLUTION:

Cramer's Rule.

Let C be the coefficient matrix of the system

$$\begin{matrix} ax + by = m \\ fx + gy = n \end{matrix} \to \begin{vmatrix} a & b \\ f & g \end{vmatrix}.$$

The solution of the system is

$$x = \frac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|} \text{ and } y = \frac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|} \text{ if } C \neq 0.$$

$$C = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}$$

$$|C| = (2)(2) - (-1)(1)$$
$$= 5$$

$$x = \frac{\begin{vmatrix} -9 & -1 \\ 8 & 2 \end{vmatrix}}{5} \text{ and } y = \frac{\begin{vmatrix} 2 & -9 \\ 1 & 8 \end{vmatrix}}{5}$$

$$x = \frac{(-9)(2) - (-1)(8)}{5} \text{ and } y = \frac{(2)(8) - (-9)(1)}{5}$$

$$x = \frac{-10}{5} \text{ and } y = \frac{25}{5}$$

$$x = -2 \text{ and } y = 5$$

Therefore, the solution of the system is (–2, 5).

22. $x - y + 2z = 0$

$3x + z = 11$

$-x + 2y = 0$

SOLUTION:

Cramer's Rule.

Let C be the coefficient matrix of the system

$$\begin{matrix} ax + by + cz = m \\ fx + gy + hz = n \\ jx + ky + lz = p \end{matrix} \to \begin{vmatrix} a & b & c \\ f & g & h \\ j & k & l \end{vmatrix}.$$

The solution of the system is

$$x = \frac{\begin{vmatrix} m & b & c \\ n & g & h \\ p & k & l \end{vmatrix}}{|C|}, y = \frac{\begin{vmatrix} a & m & c \\ f & n & h \\ j & p & l \end{vmatrix}}{|C|} \text{ and } z = \frac{\begin{vmatrix} a & b & m \\ f & g & n \\ j & k & p \end{vmatrix}}{|C|} \text{ if } C \neq 0.$$

$$C = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ -1 & 2 & 0 \end{vmatrix}$$

$$|C| = 11$$

$$x = \frac{\begin{vmatrix} 0 & -1 & 2 \\ 11 & 0 & 1 \\ 0 & 2 & 0 \end{vmatrix}}{11}$$
$$= \frac{44}{11}$$
$$= 4$$

$$y = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 3 & 11 & 1 \\ -1 & 0 & 0 \end{vmatrix}}{11}$$
$$= \frac{22}{11}$$
$$= 2$$

$$z = \frac{\begin{vmatrix} 1 & -1 & 0 \\ 3 & 0 & 11 \\ -1 & 2 & 0 \end{vmatrix}}{11}$$
$$= \frac{-11}{11}$$
$$= -1$$

Therefore, the solution of the system is (4, 2, –1).